Homework 4, due Monday, May 25 at 3:00pm

Instructions: Handwritten homeworks should be submitted at the beginning of class on May 25. Show all work and **explain all answers**. Answers should be clear and organized. All sheets of paper should be stapled together. No late homework will be accepted.

Write your own name in Chinese and in pinyin and include your PKU student ID number. Some questions may not be covered in class. The online textbook may be helpful: https://openstax.org/details/books/astronomy

1. Seasons, Lunar Phases, and the Earth (35 pts): This is basic astronomy that everyone should know.

(a) What causes the seasons? Draw a picture.

(b) What causes the lunar phases? Draw a picture.

(c) Do any planets have phases? If so, which ones?

(d) Why are some solar eclipses total eclipses and others not total? What are lunar eclipses? Are solar eclipses or lunar eclipses more common?

(e) Compare four photos of sunsets (or sunrises) from the same location at different times of the year (ideally you have taken these photos; if not, do the best you can for a minor point deduction). What time do they occur? How does the location of the sunset change? Draw a picture to explain these differences.

(f) Is the dark side of the moon ever visible from Earth, or has it only been seen by satellites and astronauts who have been to the moon?

(g) Is the dark side of the moon ever visible from Earth, or has it only been seen by satellites and astronauts who have been to the moon?

(h) The moon is moving away from Earth at 3.8 cm/yr. Approximately what year will the last total solar eclipse occur?

(i) The earth's rotation is slowing by about 0.01 seconds each year. If the earth's rotation continues to slow at this rate, would the earth day ever be the same as the earth year? If so, when would this occur?

(j) How are (h) and (i) related to each other? Explain in detail.

2. Searching for Life in the Universe (25 pts)

(a) If you could know everything about a planet and its atmosphere, what do you think would be the most important properties to understand whether the planet is habitable? How would you define whether a planet is habitable?

(b) What observations would you want to do to determine habitability? What about life?

(c) Why is that search for life not currently feasible?

(d) The equilibrium temperature, T_{eq}, of the Earth depends on how much energy is absorbed, as

$$T_{\text{eq}}^4 = \frac{L_\odot (1-a)}{16\pi d^2 \sigma} \tag{1}$$

where the Solar Luminosity $L = 3.83 \times 10^{26}$ W m^2, the distance d from the Sun to the Earth is 1 AU, a is albedo (how much energy is reflected), and Stefan-Boltzmann constant is σ is 5.67×10^{-8} W m^2 K^4. Assuming $a = 0.3$, what is the equilibrium temperature of the Earth?

(e) As discussed previously, the luminosity of a star on the main sequence is roughly proportional to the mass to the fourth, $L \propto M^4$. If a planet around a 5 $M_\odot$ star has the same equilibrium temperature of the Earth, how far from its host star would it be located? What would the distance be if the planet were around a 0.2 $M_\odot$ star?

(f) Is there anywhere in our solar system (besides Earth) where it makes sense to look for life?

3. How many stars that have been born are still here? (40 pts): In this question, we will calculate how many stars that have been born in the Milky Way are still here. We can only answer this question with simplifying (and incorrect) assumptions, but these should lead us to a reasonable answer. For the purposes of this question, assume that stars have masses between $0.1 - 100$ M$_\odot$.

This is a difficult question that requires some math and could be done with excel, a programming language, or analytically. I expect that most people will unable to complete this question.

(a) The history of star formation is measured by averaging the rate of star formation in galaxies across cosmic time. The generally accepted star formation rate, SFR is:

$$SFR = 0.015 \frac{(1+z)^{2.7}}{1 + [(1+z)/2.9]^{5.6}}, \tag{2}$$

where SFR is in units of $M_\odot$ yr^{-1} Mpc^{-3} and z is redshift. Plot the star formation rate versus redshift, from $z = 0 - 10$, and separately the star formation rate versus lookback time, from now through 13 billion years ago. Estimate the lookback time $t(z)$ as

$$t(z) = \frac{2}{3H_0}(1 - \frac{1}{(1+z)^{3/2}}, \tag{3}$$

where the Hubble constant is $H_0 = 70$ km/s/Mpc.

(b) The *initial mass function* describes the distribution of stars that form versus stellar mass. A commonly used IMF from Salpeter et al. (1955) is:

$$\xi(m)\Delta(m) = \xi_0 (m/M_\odot)^{-2.35} (\delta m/M_\odot), \tag{4}$$

where $\xi(m)\Delta(m)$ is the number of stars per mass interval. In other words, this means that the distribution of stars has a power-law slope of -2.35, so there are more low-mass stars than high-mass stars. Plot the IMF from $0.1 - 100$ M$_\odot$ and use this mass range for the rest of Question 4. For every one star from 90–100 M$_\odot$, how many stars are there from $0.9 - 1.0$ M$_\odot$? What fraction of stars are > 8 M$_\odot$, the approximate cutoff for supernova?

(c) The luminosity of a star can be estimated as

$$L/L_\odot \approx (M/M_\odot)^{3.9}. \quad (5)$$

Assume that the lifetime of the sun is 10 billion years (10^{10} yr), after which H burning in the core will cease. Plot the stellar lifetime versus mass. This question is similar to a question in homework 3.

(d) Combine your answers to (b) and (c) to estimate the fraction of stars that formed 8 billion years ago that have burned out. Do the same for stars born 2 billion years ago.

(e) Now expand your answer to (d) by plotting the fraction of stars that have burned out versus lookback time. This can be done through brute force, by calculating answers at a range of ages and summing them up, or analytically. Brute force is best done either with a programming language or advanced use of excel, though it can be done more simply with excel.

(f) Combine the plot from (e) with the star formation history from (a) to estimate the fraction of stars that have ever been born in the Milky way are still with us today.

(g) Assuming that the star formation rate versus cosmic time from (a) continues into the future, what fraction of all stars that will ever formed have already formed?

(h) In a galaxy in the Virgo cluster (located at 16.5×10^6 pc from earth), an intelligent race of Virgoans appeared at the same time as Earth. Their astronomers are looking at our local group of galaxies. The Milky Way has an absolute magnitude of -20.5 in the V band. Approximately how bright is the Milky Way in the V band?

(i) If the Milky Way stopped forming stars today, when would it fade from view of the Virgoans? Assume that their capabilities limit them to the V-band and that they are sensitive to V=30 mag, similar to deep images obtained with Hubble Space Telescope, and that the Milky Way would eventually be a point source.

(j) What do you think the Virgoans look like? Draw a picture or ask for a cartoon from your favorite LLM.